Real Demography and Pension System Sustainability

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MAF 2010 - Ravello
7- 9 aprile 2010

SUMMARY

Introduction

This paper analyzes the issue of pension system sustainability in a logical mathematical key with great relevance to the rules of the demographic aspects.

A mathematical formalization of a defined contribution pension scheme in a partially funded framework is given. In this framework a demographic representation of pension system collectivity, not based on fictitious assumptions but founded on actual reality, is used.

Logical and mathematical conditions of sustainability, which are deduced by evolution equations of differential type, are given to control the pension system financial state.

A pension system is **logically sustainable** (Angrisani, 2008) if the following phases are defined.

1. The rules phase

   Rules for contribution and for pension benefits calculation are fixed.

2. The control phase

   Well-founded logical and mathematical rules and indicators for controlling the financial sustainability of a pension system are defined.

3. The re-equilibrium phase

   When the pension system has to be brought back to the sustainability levels fixed in the second phase, re-equilibrium has to be activated on the bases of control variables and predetermined modalities.

Now in literature

Notional Defined Contribution pension systems

The Swedish Pension System (*Inkomstpension*):

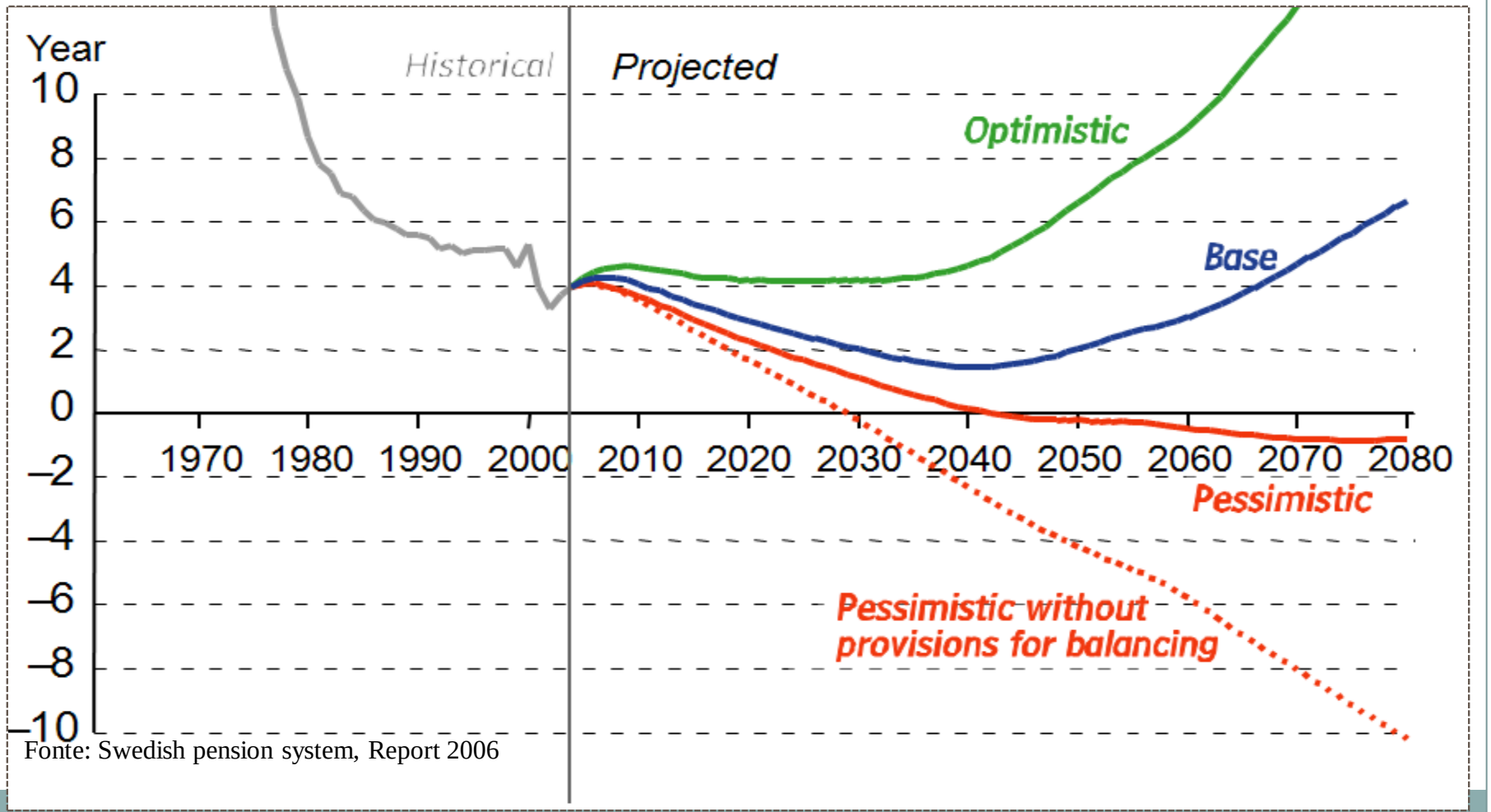
- Constant contribution rate
- The rate of return on pension credit is the growth rate of average wage
- Annuity divisor specific for cohorts
- Buffer Fund
- Sustainability indicator defined Balance Ratio $BR(t) = \frac{C(t)TD(t) + BF(t)}{PL(t)}$
- Sustainability condition

BR
- >1 → *Assets greater than liabilities*
- $=1$ → *Equilibrium*
- <1 → *Liabilities exceed assets and balancing is activated*

But…

Fund Strength

Size of buffer fund divided by pension disbursements in the same year



Fonte: Swedish pension system, Report 2006

The theoretical framework of Swedish Pension System Reform is based on

Settergren, O., & Mikula, B. (2005). The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest. *Journal of Pension Economics and Finance* , *4* (02), 115-138.

- **The sustainability is not assured in a logical key.**

- **Steady State assumption is used but it does not seem very realistic particularly under the demographic profile.**

"*…The method described in this study appears to be a powerful tool for assessing the performance of PAYG pension systems. However, questions remain, and we will need to see … a more complete analysis, before fully understanding what is being proposed for the rate of return* ***out of steady state…***

While the literature…is certainly highly relevant, ***most of it takes a comparative steady state approach and does not deal with nonsteady…" .***

(R. Lee, Discussion of "The rate of Return of Pay-As-You-Go Pension Sistems: A more Exact Consumption- Loan Model of Interest" in Holzmann, R., & Palmer, E. (2006). *Pension reform: Issues and prospects for non-financial defined contribution (NDC) schemes.* World Bank Publications.

Model Structure

A defined contribution pension system is considered.
The financial pension system management is of PAY-AS-YOU-GO type with a funded component. We refers to it as Partially Funded Defined Contribution (PFDC) pension system .

We represent the demography of the pension system without predetermined assumptions through a model in continuous time using functions of two real variables, age x and time t.
We start from very general density functions such that they can represent each type of demographic reality.